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Wave propagation through non-uniform plasma

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The dispersion relation for the waves which propagate through a non-uniform plasma has been deduced. The main purpose of this is to find the effect of density gradient on the wave propagation. It is found that density gradient may give rise to instability of longitudinal and transverse waves, provided the stream (or thermal) velocity is present. The dependence of instability factor on the density gradient has been discussed in some details. The instabilities of the waves (longitudinal and transverse) due to density gradient have been found to be significant in the rarified medium having large density gradient. Some hints of the possible applications to the present analysis have also been discussed.

1. INTRODUCTION

The small density gradient in a plasma medium may play an important role in the phenomenon like coupling between longitudinal plasma waves and transverse electromagnetic waves (Chakraborty 1970, 1971, 1973). Application of the theory of coupling in the absence or presence of streams to explain the solar bursts have been made by many authors (Field 1956, Ginzburg & Zheleznyakov 1964, Yip 1970). The problem of stream instabilities has been discussed by some authors (Clemmow & Dougherty 1969). In an earlier paper (Paul & Bandyopadhaya 1972) we studied the wave propagation in a plasma medium having density gradient without consisting of streams. We calculated there the amount of density variation due to electromagnetic wave propagation. Here we propose to study the wave propagation in a plasma having small density gradient in the presence of electron and ion streams. The medium, however, has been assumed not to have any temperature gradient. This, however, has been considered in a different paper (Bandyopadhaya & Paul 1973).

The mathematical technique of this investigation is a usual one (Tenenbaum 1967, Bondyopadhaya & Paul 1973). However, after finding out the dispersion relation we have considered the particular cases of it. Some important results concerning the rate of density gradient in the instability phenomenon of longitudinal as well as transverse wave in the presence of streaming (or thermal) motion of constituent particles, have been discussed.

2. Assumptions and Basic Equations

We consider a fully ionised plasma having small density gradient, and consisting of electron and ion streams, under the influence of static and spatially uniform magnetic field $\boldsymbol{H}_0$. We assume that,

i) before perturbation the medium was at equilibrium,

ii) gravitational force, viscous force, and force due to collision are negligibly small compared with other forces present,

iii) motions of the particles are non-relativistic,

iv) temperature is uniform throughout the medium,

v) the variables in the medium are perturbed as

$$\boldsymbol{E} = 0+\boldsymbol{E}, \quad \mathrm{H} = \boldsymbol{H}_0+\boldsymbol{H}, \quad \boldsymbol{u} = \boldsymbol{U}+\boldsymbol{u}, \quad n = N+n$$

where the first and second terms in r.h.s. are respectively unperturbed and perturbed values,

vii) the space variation of the perturbed quantities are proplitional to $\exp(iKz)$, where K is the wave number.

viii) the time variation of the perturbed quantities are proportional to $\exp.(-i \int \omega dt)$, where ω (the wave frequency) is a function of time (vide appendix),

ix) the phase velocity (ω/K) is much greater than the microscopic velocity ($\boldsymbol{u}$),

x) the perturbed quantities are much smaller than the unperturbed quantities,

xi) the unperturbed velocity (i.e., stream velocity U) is constant in space and time,

xii) the unperturbed density (N) has gradient in the direction of wave propagation.

All these assumptions are usually made for the study of linear wave propagation in plasma. The basic linearised equations which govern this wave propagation are (Zheleznyakov 1970)

$$i\omega_s'\boldsymbol{u}_s' - iKV_s^2(n_s/N_s)I_z + (q_s/m_s)\{\mathrm{E}+c^{-1}[\boldsymbol{U}_s\times\boldsymbol{H}]+c^{-1}[\boldsymbol{u}_s\times\boldsymbol{H}]_o\} = 0, \quad \ldots \quad (1)$$

$$n_s = (N_s/\omega_s')(K-iK_0)u_{sz}, \quad \ldots \quad (2)$$

$$[\nabla\times\boldsymbol{H}] = c^{-1}(\partial\boldsymbol{E}/\partial t)+4\pi c^{-1}\boldsymbol{j}, \quad \ldots \quad (3)$$

$$[\nabla\times\boldsymbol{E}] = -c^{-1}(\partial\boldsymbol{H}/\partial t), \quad \ldots \quad (4)$$

$$(\nabla\cdot\boldsymbol{E}) = 4\pi Q, \quad \ldots \quad (5)$$

$$(\nabla\cdot\boldsymbol{H}) = 0, \quad \ldots \quad (6)$$

where $Q = (n_1 - n_2)e$; $\boldsymbol{j} = \{(N_1\boldsymbol{u}_1 + n_1\boldsymbol{U}_1) - (N_2\boldsymbol{u}_2 + n_2\boldsymbol{U}_2)\}e$; $V_s = \sqrt{\chi T/m_s}$, the thermal velocity (χ being Boltzmann constant), $\omega_s' = (\omega - KU_{sz})$, I_z is the unit vector along z-axis, $S = 1$, for ion component, $S = 2$, for electron component, $\nabla N_s = N_s/L = K_0 N_s$ (say) $=$ constant, where $K_0 = L^{-1}$, L being the characteristics length of basic density variation and the other terms have their usual meaning.

3. Dispersion Relations (D.R.)

The equations from which dispersion relations can be obtained are,

$$(K^2c^2 - \omega^2 - i\partial\omega/\partial t)\boldsymbol{E} - K^2c^2E_zI_z - 4\pi ie[N_1(\omega - iK_0U_{1z})\boldsymbol{u}_1 - N_2(\omega - iK_0U_{2z})\boldsymbol{u}_2]$$

$$-4\pi ie(K - iK_0)[N_1/\omega_1')(\omega - iK_0U_{1z})u_{1z}\boldsymbol{U}_1 - (N_2/\omega_2')(\omega - iK_0U_{2z})u_2\boldsymbol{U}_2]$$

$$-4\pi e(K - iK_0)[(N_1/\omega_1'^2)(\partial\omega/\partial t)u_{1z}\boldsymbol{U}_1 - (N_2/\omega_2'^2)(\partial\omega/\partial t)u_{2z}\boldsymbol{U}_2] = 0 \qquad \ldots (7)$$

and

$$i\omega_s'\boldsymbol{u}_s - iKV_s^2/w_s')(K - iK_0)u_{sz}I_z + (q_s/m_s)\{\mathrm{E} - (i/\omega)[\boldsymbol{U}_s \times [\nabla \times \mathrm{E}]] + c^{-1}[\boldsymbol{u}_s \times \boldsymbol{H}_0]\} = 0. \qquad \ldots (8)$$

Eq. (7) has been obtained from eqs. (3) and (4) with the help of continuity eq. (2). Eq. (8) has been obtained from eqs. (1) with the help of eqs. (2) and (4).

It is observed from eqs. (7) and (8) that the components of magnetic field $\boldsymbol{H}_0$ and stream velocity $\boldsymbol{U}_s$ which are perpendicular to the wave propagation, are responsible for coupling between longitudinal and transverse waves. We, however, are interested in the uncoupled wave propagation. For this reason, let us take

$$\boldsymbol{U}_s \equiv (0, 0, \boldsymbol{U}_s), \qquad \boldsymbol{H}_0 \equiv (0, 0, \boldsymbol{H}_0),$$

i.e., both the stream velocity and magnetic field have component only in the direction of wave propagation. Under these circumstances $\boldsymbol{E}$ and $\boldsymbol{U}_s$ eliminant of eqs. (7) and (8) leads to the following dispersion relations

$$\omega^2 \pm i(K_0/2)(\omega_{p1}U_{1z} + \omega_{p2}U_{2z})$$

$$-\frac{\omega_{p1}^2[(\omega - iK_0U_{1z})\{\omega_1' + (K - iK_0)U_{1z}\} \pm i(K_0/2)(K - iK_0)(\omega p_1/\omega_1')U_{1z}^2]}{[\omega_1'^2 - K(K - 2iK_0)V_1^2]}$$

$$-\frac{\omega_{p2}^2[(\omega - iK_0U_{2z})\{\omega_2' + (K - iK_0)U_{2z}\} \pm i(K_0/2)(K - iK_0)(\omega p_2/\omega_2')U_{2z}^2]}{[\omega_2'^2 - K(K - 2iK_0)V_2^2]} = 0, \qquad \ldots (9)$$

and

$$K^2c^2 - \omega^2 \pm i(K_0/2)(\omega p_1U_{1z} + \omega p_2U_{2z})$$

$$+\frac{\omega p_1^2\omega_1'(\omega - iK_0U_{1z})(\omega_1' \pm \Omega_{1z}}{\omega(\omega_1'^2 - \Omega_{1z}^2)} + \frac{\omega p_2^2\omega_2'(\omega - iK_0U_{2z})(\omega_2' \mp \Omega_{2z})}{\omega(\omega_2'^2 - \Omega_{2z}^2)} = 0 \qquad \ldots (10)$$

for longitudinal and transverse wave respectively, where we have used,

$$\frac{\partial}{\partial t}\,\omega) = \pm\frac{\partial}{\partial t}(\omega p_1+\omega p_2) = \pm\ \frac{\omega p_1}{2}\cdot\frac{1}{N_1}\cdot\frac{\partial N_1}{\partial t} \pm \frac{\omega p_2}{2}\cdot\frac{1}{N_2}\frac{\partial N_2}{\partial t}$$

$$= \pm(K_0/2)(\omega p_1 U_{1z}+\omega p_2 U_{2z}), \quad [\text{vide Appendix}].$$

$$\omega p_1{}^2 = 4\pi N_1 e^2/m_1, \qquad \omega p_2{}^2 = 4\pi N_2 e^2/m_2,$$

$$\Omega_1 = e\boldsymbol{H}_0/m_1 c. \qquad \Omega_2 = e\boldsymbol{H}_0/m_2 c.$$

ω_{p_1} and ω_{p_2} being the plasma frequencies, Ω_1 and Ω_2 being the gyrofrequencies of the ion and electron components respectively. In this connection it may be pointed out that D.R. (9) and (10) do not hold if $(\omega-KU_s)^2-\Omega_s{}^2=0$ and $(\omega-KU_s)=0$ (for all S) respectively, because under the above conditions the solution for $\boldsymbol{u}_s$ can not be obtained from eq. (8).

4. Discussion

It is easy to see from D.R. (9) and (10) that density gradient, is capable to affect the propagation of longitudinal and transverse waves. Let us discuss nature of propagation of the waves, under different conditions.

A. *Instability of Longitudinal Wave*

(i) *Instability of cold non-uniform plasma in the presence of streams* : Let us suppose that the plasma is cold (i.e., $V_1 = V_2 = 0$) and the stream velocities of ions and electrons have the same magnitude i.e., $U_1 = U_2 = U$ (say). Therefore, the longitudinal D.R. reduces to

$$\begin{aligned}\omega^2 &\pm iK_0U(\omega p_1+\omega p_2)-(\omega p_1{}^2+\omega p_2{}^2)(\omega-iK_0U)^2/(\omega-KU)^2\\ &\pm K_0(K-iK_0)U^2(\omega p_1{}^3+\omega p_2{}^3)/2(\omega-KU)^3 = 0. \qquad \ldots (11)\end{aligned}$$

This expression shows the unstable nature of longitudinal wave due to non-uniformity (in density). To have a clear picture of this instability let us impose some restrictions on the values of K and K_0, as

(i) $|K| >> |iK_0|$ i.e., $\lambda << L$,

(ii) $|K| \sim |iK_0|$ i.e., $\lambda \sim L$,

where λ and L are respectively the wave length of the propagating wave and the characteristic length of density variation. The other case, namely $|K| << |iK_0|$ i.e., $\lambda >> L$ will not be considered, because in this case Fourier analysis will be invalid. However, we shell consider the spatial instability problem only.

Let us consider the case (*i*) $\lambda << L$. The solution for K (obtained from eq. (11)) is given by

$$KU = \left[\omega \mp (\omega p_1{}^2 + \omega p_2{}^2)^{\frac{1}{2}} \left\{1 - \frac{K_0{}^2 U^2 (\omega p_1 + \omega) p_2}{4\Lambda^3}\right\}\right]$$

$$i \left[\frac{K_0 U}{\omega} \cdot (\omega p_1{}^2 + \omega p_2{}^2)^{\frac{1}{2}} \left\{1 - \frac{(\omega p_1 + \omega p_2)}{4\Lambda}\right\}\right], \qquad (12)$$

where we have assumed

$$|iK_0 U(\omega p_1 + \omega) \quad \text{and} \quad (\omega - KU) \qquad \omega p_1, \omega p_2.$$

From eq. (12) it is observed that the instability factor (imaginary part of K) involves density gradient. And, further the instability factor must vanish with the vanishing of the streams and the density gradient. Thus we are led to conclude that if there exists any instability in cold plasma it must be due to simultaneous presence of stream and non-uniformity of the medium.

However, to understand the instability more clearly we write the instability factor as

$$\beta = \pm(2e/\omega)(\sqrt{\pi/m_1} + \sqrt{\pi/m_2})\,|\Delta N|/\sqrt{N}. \qquad \ldots (13)$$

assuming

(a) $$\omega >> \omega p_1, \omega p_2$$

i.e., the frequency of the propagating wave is much greater than the plasma frequency. This is a reasonable condition for the propagating wave

and

(b) $$N_1 = N_2 = N.$$

i.e., at equilibrium the number densities of ion and electron components are identical. This is quite reasonable for the initial equilibrium state of the ionised medium.

Expression (13) shows that the instability will be very much significant in the medium having low density but high density gradient.

Let us see how the instability varies with the density and the characteristic length of density variation. We write

$$|\nabla N|/\sqrt{N} = \sqrt{N}/L = \beta_0 \quad \text{(say)},$$

then

$$\beta \propto \beta_0.$$

Let us draw the graph showing qualitative nature of β_0 (figure 1). This figure clearly shows the variation of β_0 with both the basic density and the characteristic length of variation of basic density. The interesting nature which is worthwhile to mention is that as the density increases the differencdes between the instability curves for different characteristic length increase. This means that the variation of characteristic length is capable to produce more effect on the instability in the medium of high density than in the medium of low density. It is also observed from the figure that the slope of the curve for small characteristic length is greater than that for the large characteristic length. This indicates that the rate of change of instability for small characteristic length is higher than that for the large characteristic length.

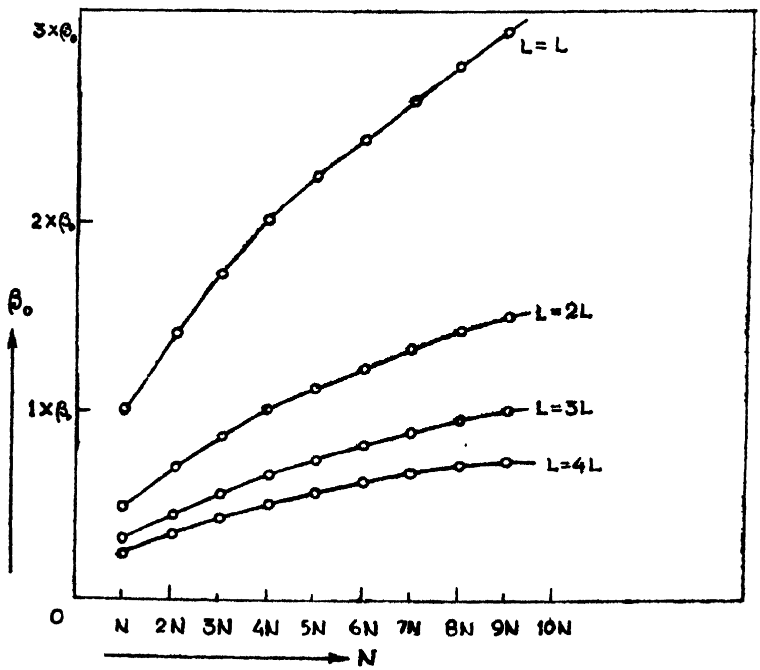

Fig. 1

We now consider less important case (ii) $\lambda \sim L$. The solution for K obtained from eq. (11) is

$$KU = \left[\omega - \frac{(\omega p_1{}^2+\omega p_2{}^2)}{\omega}\left\{1 \pm \frac{K_0{}^2U^2(\omega p_1+\omega p_2)}{2\omega^3}\right\}\right]$$

$$\pm i\left[\frac{(\omega p_1{}^2+\omega p_2{}^2)}{\omega^2}K_0U\left\{1-\frac{(\omega p_1+\omega p_2)}{2\omega}\right\}\right] \quad \dots (14)$$

assuming

$$|\omega^2| > |iK_0U(\omega_{p1}+\omega_{p2})|.$$

The above solution shows that in this case also the instability arises only due to simultaneous presence of the non-uniformity and the stream. Here the instability factor reads

$$\beta' = \pm\frac{4\pi e^2}{\omega^2}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)|\nabla N| \quad \dots (15)$$

assuming $\omega >> \omega_p, \omega_{p2}$ (justified for a propagating wave) and $N_1 = N_2 = N$. (justified for the equilibrium state of the ionised medium). Therefore, the longitudinal wave of wave-length $\lambda \sim L$ will be highly unstable in the medium having large density gradient.

Let us now write,

$$|\nabla N| = N/L = \beta_0' \text{ (say)}, \quad \text{therefore} \quad \beta' \propto \beta_0'.$$

This shows that for a fixed characteristic length the instability of longitudinal wave is more significant in the denser medium than in the rarified medium. However, comparison with previous case reveals that the instability of waves $\lambda \sim L$ is more sensitive to density than that of the waves with wave length $\lambda << L$. The qualitatve nature of β_0' for different values of number densities and characteristic lengths is however, understood clearly from figure 2.

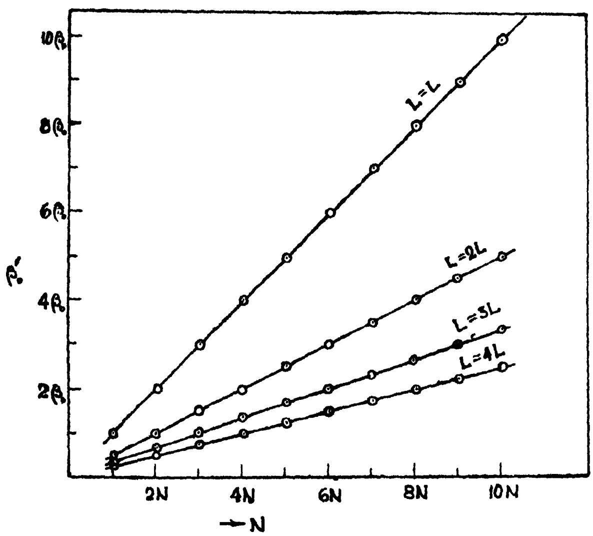

Fig. 2

(ii) *Instability of hot non-uniform plasma in the absence of streams*: In this section we shall consider the effect of non-uniformity on the instability of longitudinal wave propagating through hot plasma. We shall suppose that the streams are absent (i.e., $U_1 = U_2 = 0$) so that D.R. (9) reduces to

$$1 = \frac{\omega_{p1}^2}{[\omega^2 - K(K-2iK_0)V_1^2} + \frac{\omega_{p2}^2}{[\omega^2 - K(K-2iK_0)V_2^2]} \tag{16}$$

From eq. (16) it is observed in general, that in a streamless hot medium the longitudinal plasma wave may suffer from instability due to the presence of density gradient. To have a better insight, similar to cold plasma we consider two cases namely (i) $|K| >> |iK_0|$ i.e., $\lambda << L$ and (ii) $|K| \sim |iK_0|$ i.e., $\lambda \sim L$.

Now it can be observed directly from eq. (16) that the longitudinal wave of wave length much less than the characteristic length (i.e., $\lambda << L$) will be affected by thermal velocity and not by the density gradient. If the wave length is twice of the characteristic length (i.e., $\lambda = 2L$), then both the effects of positive density gradient and the thermal velocity are insignificant. But both effects could be much important as soon as the density gradient becomes negative.

But without imposing any restriction on K and K_0 it is also possible to see interesting features of instability of the longitudinal wave. Let us consider one component plasma, then eq. (16) yields

$$K(K-2iK_0)V_s^2-(\omega^2-\omega_{ps}^2) = 0, \tag{17}$$

where, either $S = 1$, for ion component

or $S = 2$, for electron component.

From eq. (17) the spatial instability factor becomes

$$\beta_s = |\nabla N_s|/N_s, \quad \ldots (18)$$

where we have assumed

$$\omega > (\omega_{ps}^2+K_0^2V_s^2)^{\frac{1}{2}}.$$

From the above expression it may be observed that the instability in hot plasma deserves much importance inside the medium of less density but high density gradient. Qualitatively, this result is almost same as that for cold streaming plasma (eq. (13)).

Now to know clearly the nature of the instability for different values of density and the characteristic length of density variation, we wrote,

$$|\nabla N|/N = 1/L = \beta''_0 \text{ (say)}, \quad \text{i.e., } \beta \propto \beta''_0.$$

This shows that the instability is affected by the characteristic length of density variation, not by the number density. Since the role of characteristic length is important in above instability, we draw a graph of β''_0 against different characteristic lengths (figure 3). It is observed from figure (3) that the rate of increase of instability is very much higher at small characteristic length than that at large characteristic length.

B. *Instability of Transverse Wave*

Let us consider the effect of magnetic field, and non-uniformity on the instability of transverse wave. We can write D.R. (10) as

$$K^2c^2 = [\omega^2-(\omega_{p1}^2+\omega_{p2}^2)]+i\left[\left(-\frac{K_0U}{\omega}\right)\{(\omega_{p1}^2+\omega_{p2}^2)\mp(\omega/2)(\omega_{p1}+\omega_{p2})\}\right], \quad \ldots (19.1)$$

when the magnetic field is so weak that $\Omega_s << (\omega - KU_s)$. Or, we can write eq. (10) as

$$K^2c^2 = [\omega^2 \pm (\omega - KU)(\omega_{p1}^2/\Omega_1 + \omega_{p2}^2/\Omega_2)] \pm i(K_0U/2)(\omega_{p1} + \omega_{p2}), \quad \dots (19.2)$$

when the magnetic field is so strong that $\Omega_s >> (\omega - KU_s)$ and $\Omega_s >> \omega_{ps}$.

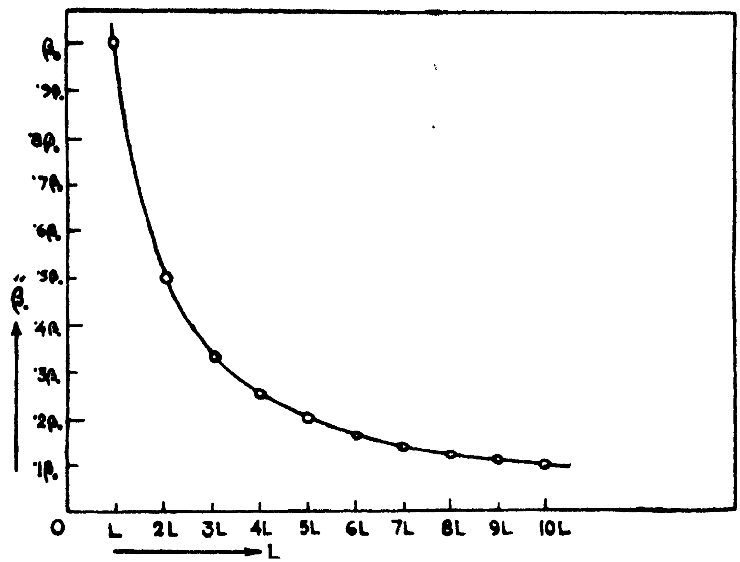

Fig. 3

In eqs. (19.1) and (19.2) we have assumed that ion and electron stream have the same magnitude (i.e., $U_1 = U_2 = U$). Expressions (19.1) and (19.2) show that similar to the instability of longitudinal wave the insteability of transverse wave arises only due to the simultaneous presence of stream and the density gradient of the particles. The instability factor in both cases is given by

$$\beta = \pm \frac{eU}{2\omega c}\left(\sqrt{\frac{\pi}{m_1}} + \sqrt{\frac{\pi}{m_2}}\right)\frac{|\nabla N|}{\sqrt{N}} \quad \dots (20)$$

assuming

$$|\omega| > |iK_0U|, \quad \omega >> \omega_{p1}, \quad \omega_{p2}, \quad N_1 = N_2 = N.$$

It is observed from eq. (20) that the instability of the transverse wave is much prominent in a rarified medium, having large density gradient. This behaviour is similar to that of longitudinal wave (eq. (13)). Therefore, in this case also, the nature of dependence of instability factor on the basis density and the characteristic length of densty variation will be same as that shown in figure 1.

From expression (20) it is to be mentioned that once the magnetic field reaches the domain of very low value or very large value, the instability of transverse wave beeome independent of magnetic field. In other words the field is able to produce the effect on the instability of transverse wave only for certain reasonable condition like $\Omega_s \approx (\omega - KU_s)$. But, in general, it is difficult to find the instability factor for such cases because D.R. in that case would involve fourth degree equation in K and fifth degree equation in ω. Hence we leave this consideration for a separate paper.

5. Phase Velocity

(A) *Longitudinal wave in cold Streaming plasma*

In general the phase velocity is given by

$$v_{ph} = \omega / R_{eal} K.$$

Therefore under the conditions (i) $\lambda << L$ and (ii) $\lambda \sim L$ the phase velocities of the longitudinal wave in cold plasma become (from eqs. (12) and (14))

$$v_{ph} = \frac{\omega U}{\left[\omega \pm (\omega_{p1}{}^2 + \omega_{p2}{}^2)^{\frac{1}{2}} \left\{ 1 - \frac{K_0{}^2 U^2 (\omega_{p1} + \omega_{p2})}{4\omega^3} \right\} \right]} \qquad \dots (21)$$

and

$$v_{ph} = \frac{\omega U}{\left[\omega - \frac{(\omega_{p1}{}^2 + \omega_{p2}{}^2)}{\omega} \left\{ 1 \pm \frac{K_0{}^2 U^2 (\omega_{p1} + \omega_{p2})}{2\omega^3} \right\} \right]} \qquad \dots (22)$$

respectively.

It is observed from eqs. (21) and (22) that phase velocity is affected by the stream and non-uniformity. Actual role of these two factors on the phase velocity is that for one mode phase velocity increases, but for the other mode phase velocity decreases. Moreover, we know, usually there does not occur any resonance for the wave having frequency much greater than plasma frequency. But we find from eqs. (21) and (22) that even for the wave having frequency $\omega >> \omega_{p1}$, ω_{p2} for one mode the presence of stream and density gradient is unable to give rise to the resonance phenomeon, but the other mode, however, may have the resonance frequency,

$$\omega_{resonance} = \begin{cases} \dfrac{\{eU|\nabla N|\}^{\frac{1}{2}}}{N^{1/4}} \cdot \left(\sqrt{\dfrac{\pi}{m_1}} + \sqrt{\dfrac{\pi}{m_2}} \right)^{1/4} \cdot \left(\dfrac{\pi}{m_1} + \dfrac{\pi}{m_2} \right)^{1/8}, & \text{for } \lambda << L. \\ \left\{ \dfrac{4e^3 U_0{}^2 |\nabla N|^2}{N^{\frac{1}{2}}} \cdot \left(\sqrt{\dfrac{\pi}{m_1}} + \sqrt{\dfrac{\pi}{m_2}} \right) \left(\dfrac{\pi}{m_1} + \dfrac{\pi}{m_2} \right) \right\}^{1/5}, & \text{for } \lambda \sim L. \end{cases}$$

(B) ***Longitudinal Wave in Hot Streamless plasma***

Now let us consider the situation where the medium is hot but does not contain any stream. In this case, it is observed from eq. (17) that the phase velocity is unaffected due to non-uniformity, when the wavelength is much smaller than the characteristic length of density variation (i.e., $\lambda << L$).

However, in general, for one component plasma we have (from eq. (17))

$$v_{ph} = \frac{\omega V_s}{[(\omega^2 - \omega_{ps}^2) - K_0^2 V_s^2]^{\frac{1}{2}}} \quad \dots (23)$$

which shows that due to the presence of non-uniformity the phase velocity of the longitudinal wave in hot medium is always increased. Moreover, the resonance frequency is given by

$$\omega_{resonance} = \left[\omega_{ps}^2 + \frac{V_s^2 |\nabla N|^2}{N^2}\right]^{\frac{1}{2}}$$

which reveals that due to non-uniformity and thermal velocity, the resonance will occur at higher frequency than plasma frequency (ω_{ps}).

(C) *Transverse Wave*

Let us now investigate what will be the effect of density gradient on the phase velocity of the transverse wave. For this, we look at expressions (19.1) and (19.2). It is observed that there is no contribution of density gradient to the real part of K. Therefore we can conclude that density gradient is incapable to affect the phase velocity for very weak or strong static magnetic field in the direction of wave propagation.

6. Possible Applications

The density wave is purely longitudinal, and electromagnetic wave is transverse in a nature. Therefore, the above discussions may be useful for the density wave theory and electromagnetic wave theory.

In the present section we shall not discuss in detail the applications of the analysis made above. Nevertheless, it appears that the present analysis could be important to a number of important astrophysical contexts.

(1) It is known that damping (Barnes 1969, Bondyopadhaya 1972) of waves in plasma are important regarding heating of astrophysical bodies. Therefore, the effect of non-uniformity of density distribution (which could produce the instability of longitudinal and transverse waves) may be incorporated in those contexts.

(2) The basic density distribution in the ionosphere above the earth is known. The gradient of this distribution may produce the density variation there (Paul & Bondyopadhaya 1972). Therefore, the ionospheric instabilities may be affected by the density gradient.

(3) The density distribution of the ionised particles above the solar photosphere is also found to have some gradient (Zholeznyakov 1964). Thus, the present theory may be considered to be worthwhile for the solar atmospheric phenomena.

(4) The density of the particles of Galactic disk is found to have large spatial gradient (Schmidt 1965, Bondyopadhaya 1974). The constituents of the materials consist of neutral as well as ionised particle (Pratap 1968). Therefore, the present analysis may be relevant to many of the Galactic instability problems. In particulars, in the central region a radial flow of ionised gases is found. This flow may be characterised by the stream velocity (U as in the present analysis) (Bondyopadhaya 1974). Thus discussions 4-A and 4-B, in particular, will be relevant to the Galactic problems associated with the radial flow from the central region of the Galaxy.

7. Remarks

1) It is to be remembered that the presence of non-uniformity and stream of the constituent particles forcibly disturb the constancy of the basic density with time. To account for this effect we have considered the time variation of the perturbed quantities to be proportional to exp.$(-i\int \omega dt)$ instead of usual one namely exp.$(i\omega t)$. (vide appendix). But we observe from our D.R. that if the perturbed solution is assumed to be exp.$(-i\omega t)$, then the instability of transverse wave due to non-uniformity vanishes, but the instability of longitudinal wave particularly in cold plasma persists. In this regard, it is worthwhile to note that the qualitative nature of instability of longitudnal wave in cold plasma remains unchanged whether ω is dependent with time or not. In otherwords it can be said that the variation of basic density with time does not affect the instability of longitudinal wave in the presence of non-uniformity and stream. Moreover discussion in Sec. 4-A-(ii) shows that the perturbed solution of the form exp.$(-i\omega dt)$ (variation of basic density with time due to presence of non-uniformity and stream) contributes nothing to the instability of hot streamless plasma. Any way, from table 1, we can have a clear picture about the role of non-uniformity in the instability of longitudinal and transverse waves under different conditions.

2) The present paper actually deals with the physics of the wave propagation and not the physical applications which, in turn, deserves separate full length discussions. However, these discussions have already been prepared and would be communicated shortly.

3) It is important to note that the density gradient has not been taken as rapidly varying quantity. Therefore, when we shall apply the present analysis we must be careful about this restriction.

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APPENDIX

From the unperturbed continuity equation, we have

$$\frac{\partial N_s}{\partial t}+U_s\frac{\partial N_s}{\partial z}=0,$$

or

$$\frac{1}{N_s}\cdot\frac{\partial N_s}{\partial t}=-\frac{U_s}{N_s}\left(\frac{\partial N_s}{\partial z}\right).$$

This shows that so long as stream velocity is present the equilibrium itself is evolving on a time scale comparable to the growth rate of the instability. Therefore, we require to perform a W.K.B. anlalysis in time i.e., the time variation of the perturbed quantities is required to assue as proportional to exp.$[-i\int\omega dt]$ instead of exp.$[-i\omega t]$, ω being the wave freqeuncy which is a function of time. Thus for example, the perturbed density

$$n\propto \exp.[-i\int\omega dt]$$

That is,

$$\frac{\partial n}{\partial t}=-i\omega n$$

and

$$\frac{\partial^2 n}{\partial t^2}=-i\frac{\partial\omega}{\partial t}n-\omega^2 n.$$

Table 1

Wave	Medium	Instability factor (β)	Conditions
Longitudinal	Cold streaming plasma medium ($U_1 = U_2 = U$)	$\beta = \pm \frac{2e}{\omega}\left(\sqrt{\frac{\pi}{m_1}}+\sqrt{\frac{\pi}{m_2}}\right)\frac{\lvert\nabla N\rvert}{\sqrt{N}}$.	$\lambda << L$; $\lvert\omega\rvert > \lvert iK_0U\rvert$; $\omega >> \omega_{p1}, \omega_{p2}$; $N_1 = N_2 = N$.
		$\beta = \pm \frac{4\pi e^2}{\omega^2}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\lvert\nabla N\rvert$	$\lambda \sim L$; $\lvert\omega\rvert > \lvert iK_0U\rvert$; $\omega >> \omega_{p1}, \omega_{p2}$; $N_1 = N_2 = N$.
	Hot streamless plasma medium.	$\beta = \frac{\lvert\nabla N\rvert}{N}$	$\omega > \left(\omega_{ps}^2 + \frac{K_0^2V_s^2}{4}\right)^{\frac{1}{2}}$ for ion component $s = 1$, electron component $s = 2$.
Transverse	Streaming plasma medium.	$\beta = \pm \frac{eU}{2\omega c}\left(\sqrt{\frac{\pi}{m_1}}+\sqrt{\frac{\pi}{m_2}}\right)\frac{\lvert\nabla N\rvert}{\sqrt{N}}$	(i) $\Omega_s >> (\omega - KU_s)$ or (ii) $\Omega_s >> (\omega - KU_s)$ and $\Omega >> \omega$. $\omega >> \omega_{p1}, \omega_{p2}$; $N_1 = N_2 = N$.

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